Topology Optimization of Electric Motor using Topological Derivative for Nonlinear Magnetostatics

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We aim at finding an optimal design for an interior permanent magnet electric motor by means of a sensitivity-based topology optimization method. The gradient-based ON/OFF method, introduced by M. Ohtake, Y. Okamoto and N. Takahashi in [1], has been successfully applied to optimization problems of this form. We show that this method can be improved by considering the mathematical concept of topological derivatives. Topological derivatives for optimization problems constrained by linear partial differential equations (PDEs) are well-understood, whereas little is known about topological derivatives in combination with nonlinear PDE constraints. We derive the topological derivative for an optimization problem constrained by the equation of nonlinear two-dimensional magnetostatics and show how this information can be used to obtain optimal designs.

Index Terms—**design optimization, permanent magnet motors, rotating machines, sensitivity analysis**

I. Introduction

Topology optimization methods originate from mechanical engineering, but have found more and more applications in electromagnetics in recent years. They aim at finding optimal designs. More precisely, they seek for the distribution of material in a design subdomain that minimizes a given design-dependent objective functional $\mathcal{J}$. The ON/OFF method was introduced in [2] and adopted to a sensitivity-based method in [1]. It is a topology optimization method that improves the design of a given device using information about the sensitivity of the cost functional with respect to a local perturbation of the magnetic reluctivity. It was shown in [3] that, in the case of a linear state equation, this sensitivity is equivalent to the mathematical concept of topological derivatives. For the optimization of electrical machines, we are faced with a nonlinear state equation. In this case, the application of the ON/OFF method generalizes directly, whereas the formula for the topological derivative has been an open problem.

In this work, we compare the ideas of the ON/OFF method and the topological derivative and show that the sensitivity used in the ON/OFF method is not the right quantity to be considered for topology optimization. We derive the formula for the topological derivative in the case of nonlinear material behavior and address implementational issues. As a model problem we consider the optimization of an interior permanent magnet (IPM) brushless electric motor, for which we want to achieve a smoother rotation. We will show numerical results obtained by the topological derivative.

II. Problem Description

We consider an IPM brushless electric motor as depicted in Fig. 1 that consists of ferromagnetic material, permanent magnets, coil areas and air regions. For our special application we do not consider any electric current induced in the coils. We aim at minimizing the total harmonic distortion (THD) of the radial component of the magnetic flux density in the air gap (see Fig. 2) by finding the optimal distribution of ferromagnetic material in the design regions of the rotor (striped areas in Fig. 1). This is realized by minimizing the distance between this radial component $\mathbf{B}_r$ and a prescribed sine curve $\mathbf{B}_r^d$ in the L^2 norm. The optimization problem looks as follows:

$$\min_{\Omega_f} \mathcal{J}(u) := \|\mathbf{B}_r(u) - \mathbf{B}_r^d\|^2_{L^2(\Gamma_0)} \tag{1}$$

$$\text{s.t.} \begin{cases} -\text{div}\ \left(\nu(|\nabla u|)\nabla u - M^\perp\right) & = 0 \quad \text{in } \Omega \\ \qquad\qquad\qquad\qquad u & = 0 \quad \text{on } \partial\Omega \end{cases} \tag{2}$$

Here, we consider the setting of two-dimensional magnetostatics where the magnetic flux density is only acting in the x_1-x_2 plane, i.e., $\mathbf{B} = (B_1, B_2, 0)^T$, and $\Omega \subset \mathbb{R}^2$. The state variable $u = u(x_1, x_2)$ denotes the third component of the magnetic vector potential, i.e., $\mathbf{B}(u) = \text{curl}\ \left((0,0,u)^T\right)$. Note that the magnetic vector potential $(0,0,u)^T$ satisfies the Coulomb gauge condition. Further, $M^\perp = (-M_2, M_1)^T$ denotes the perpendicular of the permanent magnetization which vanishes outside the permanent magnets, and ν represents the magnetic reluctivity, which is a nonlinear function $\hat{\nu}$ in the ferromagnetic subdomain Ω_f (brown area in Fig. 1) and a constant ν_0 else, i.e.,

$$\nu(|\nabla u|) = \begin{cases} \hat{\nu}(|\nabla u|) & x \in \Omega_f \\ \nu_0 & x \in \Omega \setminus \overline{\Omega}_f. \end{cases} \tag{3}$$

Note that $\mathcal{J}$ depends on Ω_f via the solution u of (2).

III. ON/OFF Method

The ON/OFF method is an efficient method for finding optimal designs in electromagnetics. The method is based on the fact that the difference between having iron or air in a spatial point is only reflected in the value of the magnetic reluctivity ν which is a constant ν_0 in the air subdomain and a nonlinear function $\hat{\nu}$ in the ferromagnetic material.

This nonlinear function $\hat{\nu}$ usually attains values that are much smaller than ν_0. The idea is to compute the sensitivity of the objective function with respect to a local perturbation of the magnetic reluctivity in one element of the finite element (FE) mesh,

$$\frac{\partial \mathcal{J}}{\partial \nu_k}, \tag{4}$$

where ν_k is the magnetic reluctivity in element k. Whenever this sensitivity is negative, assuming monotonicity of $\mathcal{J}$ with respect to ν_k, a larger value for ν_k would yield a smaller value for $\mathcal{J}$, which is realized by switching the element OFF, i.e., by setting it to air. On the other hand, if the sensitivity is positive, switching the element ON, i.e., setting it to iron, would be favorable for reducing the cost functional $\mathcal{J}$.

This method has, amongst other applications, been successfully applied to the optimization of electromagnetic shielding [1], [4] and electric motors [5]. However it is based on heuristics for the following reason: The sensitivity $\frac{\partial \mathcal{J}}{\partial \nu_k}$ just gives information about the behavior of $\mathcal{J}$ for a small variation of the magnetic reluctivity. When changing the material, the reluctivity is switched from $\hat{\nu}$ directly to ν_0 (or vice versa). The quantity to be considered for that scenario is the topological derivative.

IV. Topological Derivative

Topological derivatives were introduced in a mathematically rigorous way in [6]. The topological derivative of a domain-dependent functional $\mathcal{J} = \mathcal{J}(\Omega)$ at a point x_0 describes its sensitivity with respect to a perturbation of the domain around that point. It is defined as the quantity $G(x_0)$ satisfying a topological asymptotic expansion of the form

$$\mathcal{J}(\Omega_\varepsilon) - \mathcal{J}(\Omega) = \varepsilon^d \, G(x_0) + o(\varepsilon^d) \quad \text{as } \varepsilon \to 0. \tag{5}$$

Here, Ω_ε denotes the perturbed configuration where the material property in a small neighborhood of the spatial point x_0 is switched from iron to air or vice versa, and d is the space dimension (here: $d = 2$). It was shown in [3] that the sensitivity (4) used in the ON/OFF method is equivalent to the topological derivative in the case of a linear state equation. However, little has been known about topological derivatives in combination with nonlinear state equations (see, e.g., [7]).

We derive the topological derivative for the nonlinear problem (1)-(2) and show its advantages over the ON/OFF method. We point out that the topological derivative provides more accurate sensitivity information. Furthermore, we show that the ON/OFF method is hardly able to remove and re-introduce material in the course of an optimization process, which is possible by means of the topological derivative.

The topological derivative can be split into a linear term that resembles the sensitivities used in the ON/OFF method, and a new term which accounts for the nonlinearity of the equation. This term is neglected in the ON/OFF method. We will apply a level-set based algorithm, see [8], to find the optimal distribution of ferromagnetic material in the design regions of the electric motor introduced in Section II. Finally we will show numerical results.

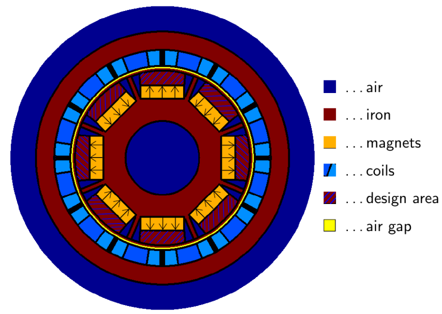

Fig. 1. Electric motor with different subdomains.

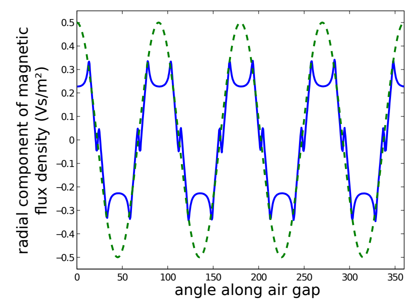

Fig. 2. Radial component of magnetic flux density $\mathbf{B}_r(u)$ along the air gap for initial design (blue) vs. desired sine curve $\mathbf{B}_r^d$ (green).

Acknowledgment

The authors gratefully acknowledge the Austrian Science Fund (FWF) for the financial support of their work via the Doctoral Program DK W1214 (project DK4) on Computational Mathematics. They also thank the Linz Center of Mechatronics (LCM), which is a part of the COMET K2 program of the Austrian Government, for supporting their work on topology and shape optimization of electrical machines.

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